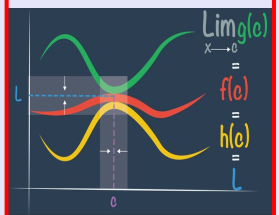
Calculus I

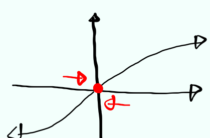
Lecture 15



Feb 19-8:47 AM

Prove $\lim_{x \to 0} \sqrt[3]{x} = 0$

$f(x) = \sqrt[3]{x}$, $a = 0$, $L = 0$ ✓



$f(x) = \sqrt[3]{x}$

For every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ whenever $|x - a| < \delta$

$|\sqrt[3]{x} - 0| < \varepsilon$ " $|x - 0| < \delta$

$|\sqrt[3]{x}| < \varepsilon$ " $|x| < \delta$

Cube both sides

$|x| < \varepsilon^3$ Pick $\delta = \varepsilon^3$

If $\varepsilon = \frac{1}{2}$, then $\delta = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

If $\varepsilon = 1$, then $\delta = 1^3 = 1$

Sep 19-7:27 AM

Evaluate $\lim\limits_{x\to -2} \dfrac{\sin(x+2)}{x^2+5x+6} = \dfrac{\sin(-2+2)}{(-2)^2+5(-2)+6} = \dfrac{\sin 0}{0} = \dfrac{0}{0}$ I.F.

Recall

$\lim\limits_{h\to 0} \dfrac{\sin h}{h} = 1$

$$= \lim_{x\to -2} \frac{\sin(x+2)}{(x+2)(x+3)} = \lim_{x\to -2}\left[\frac{\sin(x+2)}{x+2}\cdot\frac{1}{x+3}\right]$$

$$= \boxed{\lim_{x\to -2}\frac{\sin(x+2)}{x+2}} \cdot \lim_{x\to -2}\frac{1}{x+3}$$

$$= 1 \cdot \frac{1}{-2+3} = 1\cdot\frac{1}{1} = \boxed{1}$$

Sep 19-7:32 AM

Evaluate $\lim\limits_{h\to 0}\dfrac{\cos h - 1}{\sin h} = \dfrac{\cos 0 - 1}{\sin 0} = \dfrac{1-1}{0} = \dfrac{0}{0}$ I.F.

Method I: $\lim\limits_{h\to 0}\dfrac{\frac{\cos h - 1}{h}}{\frac{\sin h}{h}} = \dfrac{\overbrace{\lim\limits_{h\to 0}\frac{\cos h - 1}{h}}^{\to 0}}{\underbrace{\lim\limits_{h\to 0}\frac{\sin h}{h}}_{\to 1}} = \dfrac{0}{1} = \boxed{0}$

Recall

$\lim\limits_{h\to 0}\dfrac{\sin h}{h} = 1$ & $\lim\limits_{h\to 0}\dfrac{\cos h - 1}{h} = 0$

Method II: $\lim\limits_{h\to 0}\dfrac{\overset{(A\ -\ B)}{\cos h - 1}}{\sin h}\cdot\dfrac{\overset{(A\ +\ B)}{\cos h + 1}}{\cos h + 1}$

Recall

$\sin^2 x + \cos^2 x = 1$

$\cos^2 x - 1 = -\sin^2 x$

$$= \lim_{h\to 0}\frac{\cos^2 h - 1}{\sin h(\cos h + 1)}$$

$$= \lim_{h\to 0}\frac{-\sin^{\cancel{2}} h}{\cancel{\sin h}(\cos h + 1)}$$

$$= \lim_{h\to 0}\frac{-\sin h}{\cos h + 1}$$

Direct-Subs. $= \dfrac{-\sin 0}{\cos 0 + 1} = \dfrac{0}{1+1} = \dfrac{0}{2}$

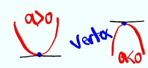
$= \boxed{0}$

Sep 19-7:37 AM

Find the x-value where $f(x) = ax^2+bx+c,\ a\neq 0$ has a horizontal tan. line.

Parabola

$\underbrace{\text{horizontal tan. line}}_{m=0}$



vertex $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$$m = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{a(x+h)^2 + b(x+h) + c - ax^2 - bx - c}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{ax^2} + 2axh + ah^2 \cancel{+bx} + bh \cancel{+c} \cancel{-ax^2} \cancel{-bx} \cancel{-c}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(2ax + ah + b)}{\cancel{h}}$$

$$= \lim_{h\to 0} \left[2ax + \cancel{ah}^{\to 0} + b\right] = 2ax + b$$

horizontal tan. line $\to m = 0 \to 2ax + b = 0$

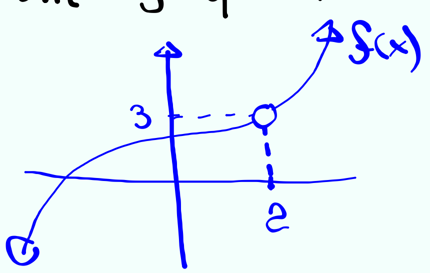
$$2ax = -b$$

$$\boxed{x = \frac{-b}{2a}}$$

Sep 19-7:46 AM

Continuity

1) From graph ⇒ No gap, No jump, No hole

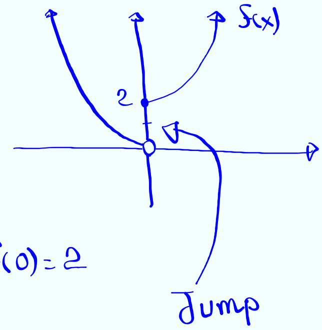


$$\lim_{x\to 2^+} f(x) = 3$$

$$\lim_{x\to 2^-} f(x) = 3 \quad \Rightarrow \lim_{x\to 2} f(x) = 3$$

$f(2)$ does not exist. $f(x)$ is not cont. at $x=2$.

Sep 19-7:57 AM



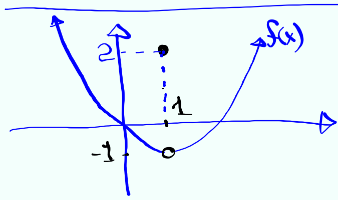
$\lim_{x \to 0^+} f(x) = 2$

$\lim_{x \to 0^-} f(x) = 0$

$f(0) = 2$

$\lim_{x \to 0} f(x)$ D.N.E.

$f(x)$ is not Cont. at $x = 0$



$\lim_{x \to 1^+} f(x) = -1$

$\lim_{x \to 1^-} f(x) = -1$

$\lim_{x \to 1} f(x) = 1$

$f(1) = 2$

$\lim_{x \to 1} f(x) \neq f(1)$ not Cont. at $x = 1$

Sep 19-8:00 AM

$f(x)$ is Cont. at $x = a$ if

$$\lim_{x \to a} f(x) = f(a)$$

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \neq 2 \\ 4 & \text{if } x = 2 \end{cases}$$

Is $f(x)$ Cont. at $x=2$?

$f(2) = 4$

$\lim_{x \to 2} f(x) = \lim_{x \to 2} \dfrac{x^2-4}{x-2}$ $\dfrac{0}{0}$

$= \lim_{x \to 2} \dfrac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$

$= \lim_{x \to 2} (x+2) = 4$

Since $\lim_{x \to 2} f(x) = f(2)$

then $f(x)$ is Cont. at 2

Sep 19-8:06 AM

Is $f(x)$ cont. at $x=2$?

$$f(x)=\begin{cases} \dfrac{x^3-8}{x-2} & \text{if } x\neq 2 \\ -12 & \text{if } x=2 \end{cases}$$

$\lim\limits_{x\to 2} f(x) = f(2)$ ✓ ✓

$f(2) = -12$

$$\lim_{x\to 2} f(x) = \lim_{x\to 2} \frac{x^3-8}{x-2} = \frac{0}{0}$$

$$= \lim_{x\to 2} \frac{\cancel{(x-2)}(x^2+2x+4)}{\cancel{x-2}} = \lim_{x\to 2} (x^2+2x+4)$$

$$= 2^2+2(2)+4$$

$$= \boxed{12}$$

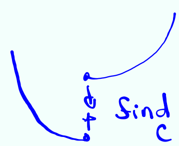
Since $\lim\limits_{x\to 2} f(x) \neq f(2)$

then $f(x)$ is not cont. at $x=2$

Sep 19-8:11 AM

Given

$$f(x)=\begin{cases} cx^2+2x & \text{if } x<2 \\ x^3-cx & \text{if } x\geq 2 \end{cases}$$



Find a value for c to make $f(x)$ cont. at $x=2$.

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2} (cx^2+2x) = c\cdot 2^2+2(2) = 4c+4$$

$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2} (x^3-cx) = 2^3-c\cdot 2 = 8-2c$$

To be cont. $\lim\limits_{x\to 2} f(x)$ must exist

$4c+4 = 8-2c$

$4c+2c = 8-4$

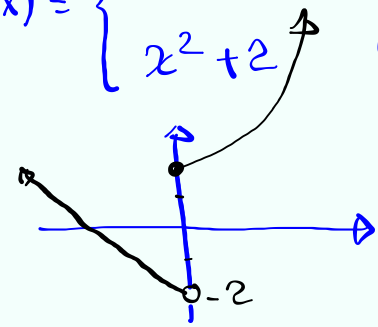
$6c = 4 \to \boxed{c=\frac{2}{3}}$

Sep 19-8:17 AM

Find a value for K such that

$$f(x) = \begin{cases} Kx - 2 & \text{if } x < 0 \\ x^2 + 2 & \text{if } x \geq 0 \end{cases}$$

become cont. at $x = 0$



$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0} (Kx - 2) = -2$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0} (x^2 + 2) = 2$$

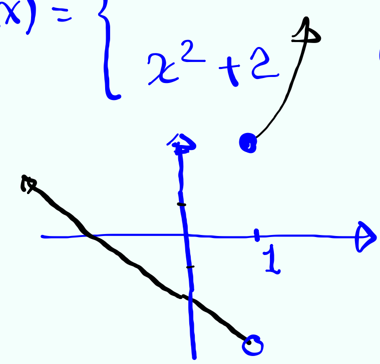
No such values

Sep 19-8:23 AM

Find a value for K such that

$$f(x) = \begin{cases} Kx - 2 & \text{if } x < 1 \\ x^2 + 2 & \text{if } x \geq 1 \end{cases}$$

become cont. at $x = 1$



$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} (Kx - 2) = K - 2$$

$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1} (x^2 + 2) = 3$$

$K - 2 = 3 \rightarrow \boxed{K = 5}$

Sep 19-8:23 AM